Rat Legs

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 All members of this group have read and approved the submission of this memo.

## **Executive Summary**

The University of Utah is engineering a brand-new roller coaster. This new rollercoaster has been themed after a rat uprising. This coaster consists of three main thrills, one main climb, one vehicle, four connecting tracks, and many supports that frame the whole ride.

To ensure rider safety around Julius Cheezer, one of the connecting tracks, Rat Legs were designed as a frame to support the entire feature with a factor of safety of 7. To achieve this safety level, 15 A-Frames were used spaced every 52 meters (Figure 1).

A close-up of a pipe

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***Figure 1:*** *Full Assembly Model of Julius Cheezer with Supporting A-Frames.*

**Introduction**

The Rat Legs support Julius Cheezer in The Rat Uprising roller coaster. Stress analysis on Julius Cheezer determines that the maximum allowable distance between supports for the track is 52 meters. From this analysis, The Rat Legs are designed to consist of 15 supports placed at 52-meter intervals along the length of Julius Cheezer.

**Components**

Julius Cheezer is a section of the track that includes a 360-degree banked turn around a 170-meter radius (Figure 2).

A close-up of a flexible tube

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| ***Figure 2:*** *Aerial view of the Julius Cheezer connecting track. The roller coaster vehicle will approach from the left when looking from the aerial view.* |

The frame that supports Julius Cheezer is an A-Frame design that consists of tubular legs ranging from 20 meters to 100 meters off the ground (Figure 3).

A metal frame with a round top

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| ***Figure 3:*** *Front view of the A-Frame supports holding a section of the track.* |

**Design**

The Rat Legs are designed to ensure that the track is supported in all loading scenarios along the length of Julius Cheezer. The allowed distance between the supports before failure occurs (either with the track or support structure) is the most important design consideration. The stress experienced at maximum loading conditions for the supports is 1923 MPa (Table 1). The stress on the track under the same conditions is 101 MPa. As the supports experience higher stress than the track, the support structure will fail before the track does. Thus, The Rat Legs design focuses on preventing failure of the supports.

The maximum stress experienced by the track and the supports is used to determine the maximum distance allowed between supports before failure occurs, with a safety factor of seven. Based on the stresses on the support structure, the maximum allowed distance between supports is 52 meters, while the maximum allowed distance between supports from the stresses on the track is 263 meters. Therefore, the distance between the supports for The Rat Legs is set at 52 meters.

***Table 1:*** *Compares the maximum stresses sustained by the track and supports through Julius Cheezer. Also included is the maximum allowable distance between supports before failure of the component occurs. The smallest of these distances determined the distance between supports in the design of The Rat Legs.*

|  |  |  |
| --- | --- | --- |
| Component | Maximum Stress  (MPa) | Allowable Distance Between Supports (m) |
| Support | 1923 | 52 |
| Track | 101 | 263 |

The design of Julius Cheezer (Table 2) is important to the design of The Rat Legs. Julius Cheezer contains three sections: entrance, spiral, and exit. The entrance is a 120 meters long straight track between the exit of Flushed Away and the spiral. As the distance between supports is 52 meters, this section has three supports. The spiral is 512 meters in length, and so requires 10 supports. The exit is an 80-meter straight track from the spiral to the entrance of Twin Towers of Rat Terror and contains two supports.

***Table 2:*** *Dimensions for the sections of Julius Cheezer. The length of each section determines how many supports are used to support that section by dividing the length of the section by the distance between supports (rounding up to the next integer)*

|  |  |  |
| --- | --- | --- |
| Section of Julius Cheezer | Length (m) | Number of Supports Needed |
| Entrance | 120 | 3 |
| Spiral | 512 | 10 |
| Exit | 80 | 2 |
| Total Track | 712 | 15 |

The Rat Legs consists of 15 A-frame supports placed at 52 m intervals along Julius Cheezer (Figure 4). The first support is placed at the beginning of Julius Cheezer (point A) and the following two supports are placed at even intervals along the entrance until the spiral. The spiral is supported by 10 A-frame supports placed between points B and C. There are two supports placed along the exit; the last support is placed at the exit of Julius Cheezer (point D).

A long shot of a metal structure

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***Figure 4:*** *The Rat Legs. There are 15 supports placed 52 meters apart along Julius Cheezer. This meets the design specification of a safety factor of seven. Point A is the entrance to Julius Cheezer, point B is the beginning of the spiral, point C is the end of the spiral, and point D is the exit of Julius Cheezer. Between points A and B there are three supports, between points B and C there are 10 supports, and between points C and D there are two supports.*

**Conclusion**

In conclusion, Rat Legs frames Julius Cheezer with a factor of safety of seven by spacing A-Frame supports 52 meters apart.

**Appendices I: Dynamic Force Analysis Methods**

## **Kinematics and Physics of Julius Cheezer**

Riders will enter Julius Cheezer before the 360-degree banked turn at approximately 44.28 meters per second and leave the connecting track at 52.28 meters per second. After entering the 170-meter radius banked turn, the vehicle and its riders will complete the entire wrap-around within 10 seconds. Travelling around the banked turn will result in centripetal force dynamics. The centripetal forces are caused by normal and tangential accelerations (Figure 1-1).

A circle with a line and a line in the middle

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***Figure 1-1:*** *Breakdown of centripetal acceleration around a curve where R is the radius of the curve, is the acceleration of the body around the curve in the tangential direction, and is the acceleration of the body around the curve in the normal direction.*

The acceleration in the tangential direction is responsible for the vehicle’s motion linearly and allows it to translate forward from the reference point of a passenger. The acceleration in the tangential direction is given by

|  |  |  |
| --- | --- | --- |
|  |  | (1) |

where is the first derivative of velocity, *t* is the time it takes to travel around the curve (seconds), *Vout* is the velocity of the vehicle leaving the connecting track (meters/second), and *Vin* is the velocity of the vehicle entering the connecting track (meters/second). The acceleration in the normal direction is responsible for the translation of the vehicle along the curve and is perpendicular to the tangential acceleration. The acceleration in the normal direction is given by

|  |  |  |
| --- | --- | --- |
|  |  | (2) |
|  |  |  |

where *V* is the velocity of the vehicle around the wrap-around (meters/second), *R* is the radius of the curve (meters), is the first derivative of length, *t* is the time it takes to travel around the curve (seconds), and *l* is the length of the wrap-around (meters).

## **Resulting Forces from Vehicle Motion on Julius Cheezer**

Using Newtons second law, the accelerations caused by the motion around the curved connecting track, Julius Cheezer, can be transformed into forces.

A diagram of a circle with arrows and a red line

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| --- |
| ***Figure 1-2:*** *Breakdown of centripetal forces around a curve where R is the radius of the curve, is the force of the body around the curve in the tangential direction, is the force of the body around the curve in the normal direction, and F is the resultant force from* . |

The acceleration in the tangential and normal directions became the forces in the tangential and normal directions respectively (Figure 1-2). This was done by utilizing Newtons second law which is given by

|  |  |  |
| --- | --- | --- |
|  |  | (3) |

where *m* is the mass of vehicle with its passengers (kilograms), and *a* is the acceleration of the vehicle around Julius Cheezer (meters/). When combining equations (1) and (2) with equation (3), the force transformation is given by

|  |  |  |
| --- | --- | --- |
|  |  | (4) |
|  |  | (5) |
|  |  |  |

where is the mass of vehicle with the passengers (kilograms), *Vout* is the velocity of the vehicle leaving the connecting track (meters/second), *Vin* is the velocity of the vehicle entering the connecting track (meters/second), *t* is the time it takes to go around the wrap-around, *l* is the length of the wrap-around (meters), and *R* is the radius of the wrap-around. Using the forces calculated in the tangential and normal directions, a resultant force can be found that is indicated in Figure 3-1 by the red vector. The magnitude of the resultant force can be calculated using the equation

|  |  |  |
| --- | --- | --- |
|  |  | (6) |

where is the force in the tangential direction given by equation (4) (Newtons), and is the force in the normal direction given by equation (5) (Newtons).

## **Stresses acting on A-Frame Supports from Dynamic Forces**

The dynamic force analysis of the vehicle along Julius Cheezer creates stresses in the A-Frame supports. Due to the orientation of the forces being horizontal and parallel to the ground, the resultant force, calculated using equation (6), creates normal bending stresses and transverse shear stresses in the A-Frames legs. The maximum transverse shear stress and maximum normal bending stress that result from the resultant force is located at the base of the A-Frame next to ground.

The maximum normal bending stress is given by the equation

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|  |  | (7) |

where *c* is the radius of the A-frame leg (meters), *M* is the maximum bending moment (Newton-meter), and *I* is the moment of inertia of the A-frame leg (. The maximum bending moment can be given by

|  |  |  |
| --- | --- | --- |
|  |  | (8) |

where *F* is the resultant force (Newtons), and *d* is the vertical distance from the ground to where the track connects to the A-frame (meters). The moment of inertia is given by

|  |  |  |
| --- | --- | --- |
|  |  | (9) |

where *d* is the diameter of the A-Frame leg (meters). After substituting equations (8) and (9) into equation (7), the maximum normal bending stress is calculated.

The equation for safety factor is given by

|  |  |  |
| --- | --- | --- |
|  |  | (10) |

where is the yield stress of the material (Pascals) and is the maximum bending stress experienced by the member (Pascals). By combining equation (7), with the desired safety factor of seven and equation (10), the distance between A-Frame supports can be calculated.

**Appendices II: Static Force Analysis Methods**

**Forces on A-Frame**

The A-frame have 2 supports that hold the total weight of the vehicle, track, and riders. A sum of the forces is used to calculate the forces of each pillar holding up the weight which is Fa and Fb. The total weight is found first. The weight is the sum of the vehicle, 26 meters of track, and max weight of riders. The equation for total weight is

|  |  |  |
| --- | --- | --- |
|  |  | (11) |

With the sum of the total weight calculated, the force of each pillar is calculated. The forces are first calculated in the x-direction. Since weight is in the y-direction, it can be ignored for now. Both Fa and Fb are in the x-direction at an angle of 20 degrees from the center. So therefore, the sum of the forces in the x-direction is

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| --- | --- | --- |
|  | . | (12) |

Since Fa and Fb are pointing in opposite directions, assuming positive x-direction is pointing right, Fb is subtracted from Fa. Solving for Fa, it’s found that Fa is equal to Fb. Now that we only have one unknown, another equation is needed. The equation is

|  |  |  |
| --- | --- | --- |
|  | . | (13) |

With this equation the forces for both pillars can be found.

**Moment and Maximum Stress**

In order to find the maximum bending moment on the track, the moment must be taken in the middle between 2 A-frame supports where the support is its weakest. At this point the track is most likely to break (Figure 2-1). This assumption considers the weight of the vehicle and its passengers as well.

A graph of a graph

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***Figure 2-1:*** *Side view of track between 2 supports along with the vehicle and riders in the middle where is most likely to break.*

A free body diagram depicting all the forces acting on the track shows that all forces except P are known (Figure 2-2). Summing the forces in the y-direction is used to calculate P.

A diagram of a diagram

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**Figure 2-2:** Free body diagram of track between 2 A-frame supports. ‘P’ is the force of each support on the track, ‘Wv’ is the weight of the vehicle, ‘Wr’ is the weight of the riders, and ‘Wt’ is the weight of the track along the track between the supports.

The equation for the sum of the forces in the y-direction is

|  |  |  |
| --- | --- | --- |
|  |  | (14) |

where P are the forces from the A-frame supports holding up the track and x is the distance between the 2 A-frame supports (Newtons). Solving for P, the new equation is

|  |  |  |
| --- | --- | --- |
|  | . | (15) |

With P found, the track is cut right in the middle to find the maximum moment (Figure 2-3). With this, the moment at the cut is found.

A diagram of a blue rectangular object with arrows pointing to the side

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***Figure 2-3:*** *Track cut in the middle between 2 A-frame supports with a free body diagram to represent forces.*

The equation for the moment at the cut of the track is

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| --- | --- | --- |
|  | , | (16) |

where Mc is the moment at the cut (Newton-meters) and P is given by equation (5). With Mc found, the maximum stress is determined. The equation is

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| --- | --- | --- |
|  | , | (17) |

where I is the moment of inertia around the neutral axis which is

|  |  |  |
| --- | --- | --- |
|  | , | (18) |

where b is the base of the track and h is the height of the track.